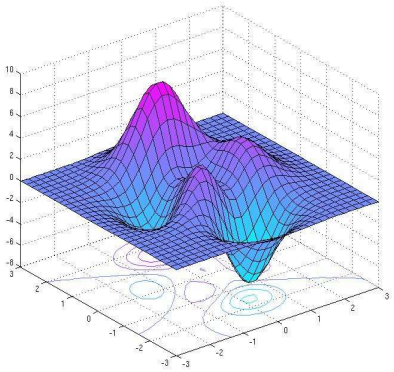
Uncertain Data in Modelling

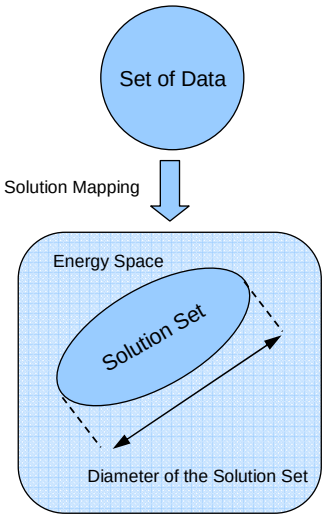
Observations ⇨ Mathematical Model ⇨ Numerical Simulations

"Mathematics is the language with which God has written the universe." Galileo

"All models are wrong, but some are useful." G. E. Box



How accurate knowledge simulations may provide?



Typically, models contain uncertain data, for example material parameters, diffusion coefficient, fish population, option volatility, etc. Thus, it is sensible to consider a set of possible data instead of single values. Our approach is non-probabilistic, we assume no other a priori knowledge except that data belongs to some interval.

Due to the uncertain data, we have to consider a set of solutions instead of a single exact solution. The diameter of the solution set is of practical interest to an analyst. It defines an *accuracy limit*. All computations dedicated to improve approximation beyond this limit make no sense. Estimates for the accuracy limit for different elliptic boundary value problems were studied in [1,2,3], where two-sided estimates for the diameter were constructed. Our methodology is based on *a posteriori functional estimates* [4,5], that provide computable lower and upper bounds for approximation error and depend explicitly on the problem data. The latter property is crucial for our analysis.

Our goal is to construct simulation tools that can help analyst to balance the errors arising from the numerical approximation and indeterminate data. In future, we plan to study hyperbolic problems.

References:

[1] O. Mali and S. Repin, *Estimates of accuracy limit for elliptic boundary value problems with uncertain data*, AMSA 2 (19), 2009.

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[3] O. Mali and S. Repin, *Estimates of the indeterminacy set for elliptic boundary–value problems with uncertain data*, J. Math. Sci. 150, 2008.

[4] P. Neittaanmäki and S. Repin, *Reliable Methods for Computer Simulation, Error Control and A Posteriori Estimates*, Elsevier, 2004.

[5] S. Repin, *A Posteriori Estimates for Partial Differential Equations*, Walter de Gruyter, 2008.

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